UK Maths Trust

Pink Kangaroo

Thursday 21 March 2024

a member of the Association Kangourou sans Frontières

supported by G-Research Overleaf

England & Wales: Year 11 or below
Scotland: S4 or below
Northern Ireland: Year 12 or below

Instructions

1. Do not open the paper until the invigilator tells you to do so.
2. Time allowed: **60 minutes**.
   No answers, or personal details, may be entered after the allowed time is over.
3. The use of blank or lined paper for rough working is allowed; **squared paper**, **calculators and measuring instruments are forbidden**.
4. **Use a B or an HB non-propelling pencil.** Mark at most one of the options A, B, C, D, E on the Answer Sheet for each question. Do not mark more than one option.
5. **Do not expect to finish the whole paper in the time allowed.** The questions in this paper have been arranged in approximate order of difficulty with the harder questions towards the end. You are not expected to complete all the questions during the time. You should bear this in mind when deciding which questions to tackle.
6. **Scoring rules:**
   5 marks are awarded for each correct answer to Questions 1-15;
   6 marks are awarded for each correct answer to Questions 16-25;
   In this paper you will not lose marks for getting answers wrong.
7. **Your Answer Sheet will be read by a machine.** Do not write or doodle on the sheet except to mark your chosen options. The machine will read all black pencil markings even if they are in the wrong places. If you mark the sheet in the wrong place, or leave bits of eraser stuck to the page, the machine will interpret the mark in its own way, or reject the answer sheet.
8. **The questions on this paper are designed to challenge you to think, not to guess.** You will gain more marks, and more satisfaction, by doing one question carefully than by guessing lots of answers. This paper is about solving interesting problems, not about lucky guessing.

Enquiries about the Pink Kangaroo should be sent to:

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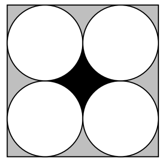
1. What is the value of $\dfrac{2 \times 0.24}{20 \times 2.4}$?

A 0.01 B 0.1 C 1 D 10 E 100

2. The figure shows a square with four circles of equal area, each touching two sides of the square and two other circles. What is the ratio between the area of the black region and the total area of the grey regions?

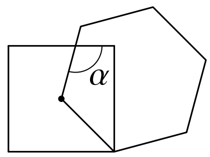


A 1 : 4 B 1 : 3 C 2 : 3 D 3 : 4 E π : 1

3. 232 and 111 are both three-digit palindromes as they read the same from left to right as they do right to left. What is the sum of the digits of the largest three-digit palindrome that is also a multiple of 6?

A 16 B 18 C 20 D 21 E 24

4. Tom draws a square. He adds a regular hexagon, one side of which joins the centre of the square to one of the vertices of the square, as shown. What is the size of angle α?

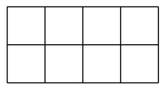


A 105° B 110° C 115° D 120° E 125°

5. Sadinie is asked to create a rectangular enclosure using 40 m of fencing so that the side-lengths, in metres, of the enclosure are all prime numbers. What is the maximum possible area of the enclosure?

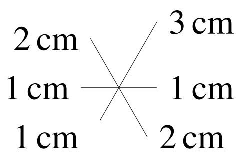
A 51 m^2 B 84 m^2 C 91 m^2 D 96 m^2 E 99 m^2

6. Lil writes one of the letters P, Q, R, S in each cell of a 2×4 table. She does this in such a way that, in each row and in each 2×2 square, all four letters appear. In how many ways can she do this?



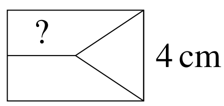
A 12 B 24 C 48 D 96 E 198

7. The diagram gives the lengths of six lines meeting at a point. Eric wishes to redraw this diagram without lifting his pencil from the paper. He can choose to start his drawing anywhere. What is the shortest distance he can draw to reproduce the figure?



A 14 cm B 15 cm C 16 cm D 17 cm E 18 cm

8. A rectangle is divided into three regions of equal area. One of the regions is an equilateral triangle with side-length 4 cm; and the other two are trapezia, as shown.



What is the length of the smaller of the parallel sides of the trapezia?

A $\sqrt{2}$ cm B $\sqrt{3}$ cm C $2\sqrt{2}$ cm D 3 cm E $2\sqrt{3}$ cm

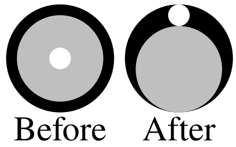
9. The ages of Jo, her daughter and her grandson are all even numbers. The product of their three ages is 2024. How old is Jo?

A 42 B 44 C 46 D 48 E 50

10. The sum of the digits of the positive integer N is three times the sum of the digits of $N + 1$. What is the smallest possible sum of the digits of N?

A 9 B 12 C 15 D 18 E 27

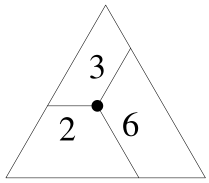
11. Polly has three circles cut from three pieces of coloured card. She originally places them on top of each other as shown. In this configuration the area of the visible black region is seven times the area of the white circle.



Polly moves the circles to a new position, as shown, with each pair of circles touching each other. What is the ratio between the areas of the visible black regions before and after?

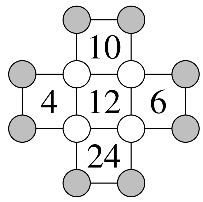
A 3 : 1 B 4 : 3 C 6 : 5 D 7 : 6 E 9 : 7

12. A point is chosen inside an equilateral triangle. From this point we draw three segments parallel to the sides, as shown. The lengths of the segments are 2 m, 3 m and 6 m. What is the perimeter of the triangle?



A 22 m B 27 m C 33 m D 39 m E 44 m

13. A number is written in each of the twelve circles shown. The number inside each square indicates the product of the numbers at its four vertices. What is the product of the numbers in the eight grey circles?



A 20 B 40 C 80 D 120 E 480

14. The sides, in cm, of two squares are integers. The difference between the areas of the two squares is $19\,\text{cm}^2$. What is the sum of the perimeters of the two squares?

A 38 cm B 60 cm C 64 cm D 72 cm E 76 cm

15. Molly has a set of cards numbered 1 to 12. She places eight of them at the vertices of an octagon so that the sum of every pair of numbers at opposite ends of an edge of the octagon is a multiple of 3.

Which numbers did Molly not place?

A 1, 5, 9 and 12 B 3, 5, 7 and 9 C 1, 2, 11 and 12
D 5, 6, 7 and 8 E 3, 6, 9 and 12

16. Peter always tells the truth or always lies on alternate days. One day, he made exactly four of the following five statements. Which one did he not make?

A I lied yesterday and I will lie tomorrow.
B I am telling the truth today and I will tell the truth tomorrow.
C 2024 is divisible by 11.
D Yesterday was Wednesday.
E Tomorrow will be Saturday.

17. Matthew rolled a normal die 24 times. All numbers from 1 to 6 came up at least once. The number 1 came up more times than any other number. Matthew added up all the numbers. The total he obtained was the largest one possible. What total did he obtain?

A 83 B 84 C 89 D 90 E 100

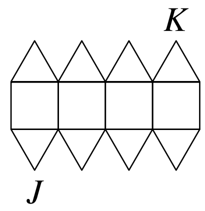
18. For some positive integer n, the prime factorisation of the number $n! = 1 \times 2 \times \ldots \times n$ is of the form shown below.

$$2 \times 3 \times 5 \times 7 \times 11 \times 13^4 \times 17 \times \text{[illegible]} \times 41 \times 43 \times 47$$

The primes are written in increasing order. Ink has covered some of the primes and some of the exponents. What is the exponent of 17?

A 1 B 2 C 3 D 4 E 5

19. Kevin makes a net using a combination of squares and equilateral triangles, as shown in the figure. The side length of each square and of each triangle is 1 cm. He folds the net up to form the surface of a polyhedron. What is the distance between the vertices J and K in this polyhedron?



A $\sqrt{5}$ cm B $(1 + \sqrt{2})$ cm C $\frac{5}{2}$ cm

D $(1 + \sqrt{3})$ cm E $2\sqrt{2}$ cm

20. Jill has some unit cubes which are all black, grey, or white. She uses 27 of them to build a $3 \times 3 \times 3$ cube. She wants the surface to be exactly one-third black, one-third grey, and one-third white. The smallest possible number of black cubes she can use is X and the largest possible number of black cubes she can use is Y. What is the value of $Y - X$?

A 1 B 3 C 6 D 7 E 9

21. Meera walked in the park. She walked half of the total time at a speed of 2 km/h. She then walked half of the total distance at a speed of 3 km/h. Finally, she completed the remainder of the walk at a speed of 4 km/h. For what fraction of the total time did she walk at a speed of 4 km/h?

A $\frac{1}{14}$ B $\frac{1}{12}$ C $\frac{1}{7}$ D $\frac{1}{5}$ E $\frac{1}{4}$

22. Given the integers from 1 to 25, Ajibola wants to remove a few and then split those that remain into two groups so that the products of the integers in each group are equal. Ajibola removes the smallest possible number of integers in order to achieve this. What is the sum of the numbers which Ajibola removes?

A 75 B 79 C 81 D 83 E 89

23. Twenty points are equally spaced around the circumference of a circle. Kevin draws all the possible chords that connect pairs of these points. How many of these chords are longer than the radius of the circle but shorter than its diameter?

A 90 B 100 C 120 D 140 E 160

24. There are n distinct lines in the plane. One of these lines intersects exactly 5 of the n lines, another of these intersects exactly 9 of the n lines, and yet another intersects exactly 11 of them. Which of the following is the smallest possible value of n?

A 12 B 13 C 14 D 25 E 28

25. Suppose m and n are integers with $0 < m < n$. Let $P = (m, n)$, $Q = (n, m)$, and $O = (0, 0)$. For how many pairs of m and n will the area of triangle OPQ be equal to 2024?

A 4 B 6 C 8 D 10 E 12